Whenever we repeatedly increase an amount by a percentage, it’s a “growth” situation. If \( r \) represents the growth rate and \( t \) represents the time that has passed:

We convert the Standard Exponential Form: 
\[ f(x) = a \cdot b^x \]

to the Exponential Growth Model:
\[ y = a \cdot (1 + r)^t \]

These two forms are essentially the same.

A painting is sold for $1800, and its value increases by 11% each year after it is sold. Find the value of the painting in 30 years.

Write the exponential growth function for this situation.

\[ y = a(1 + r)^t \]

\[ = 1800(1 + 0.11)^t \]

\[ = 1800(1.11)^t \]

Find the value in 30 years.

\[ y = 1800(1.11)^{30} \]

\[ \approx 41,206.13 \]

After 30 years, the painting will be worth approximately $41,206.
\[ y = 1800(1.11)^t \]

Create a table of values to graph the function.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( y )</th>
<th>((t, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1800</td>
<td>(0, 1800</td>
</tr>
<tr>
<td>8</td>
<td>4148</td>
<td>(8, 4148)</td>
</tr>
<tr>
<td>16</td>
<td>9560</td>
<td>(16, 9560)</td>
</tr>
<tr>
<td>24</td>
<td>22,030</td>
<td>(24, 22,030)</td>
</tr>
<tr>
<td>32</td>
<td>50,770</td>
<td>(32, 50,770)</td>
</tr>
</tbody>
</table>

The domain is the set of real numbers \( t \) such that \( t \geq 0 \).

The range is the set of real numbers \( y \) such that \( y \geq 1800 \).

The \( y \)-intercept is the value of \( y \) when \( t = 0 \), which is the value of the painting when it was sold.
A baseball trading card is sold for $2, and its value increases by 8% each year after it is sold. Find the value of the baseball trading card in 10 years.

Write the exponential growth function for this situation.

\[ y = a(1 + r)^t \]
\[ = \boxed{(1 + \boxed{0.08})^t} \]
\[ = \boxed{(1.08)^t} \]

Find the value in 10 years.

\[ y = a(1 + r)^t \]
\[ = \boxed{(1.08)^{10}} \]
\[ = \boxed{2.1609} \]

After 10 years, the baseball trading card will be worth approximately $\boxed{2.16}$. 
Create a table of values to graph the function.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$y$</th>
<th>$(t, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>$(0, 2)$</td>
</tr>
<tr>
<td>3</td>
<td>2.52</td>
<td>$(3, 2.52)$</td>
</tr>
<tr>
<td>6</td>
<td>3.17</td>
<td>$(6, 3.17)$</td>
</tr>
<tr>
<td>9</td>
<td>4.00</td>
<td>$(9, 4.00)$</td>
</tr>
<tr>
<td>12</td>
<td>5.04</td>
<td>$(12, 5.04)$</td>
</tr>
</tbody>
</table>

The domain is the set of real numbers $t$ such that $t \geq 0$.

The range is the set of real numbers $y$ such that $y \geq 2$.

The $y$-intercept is the value of $y$ when $t = 0$, which is the value of the card when it was sold.
A savings account with an initial balance of $1000 earns 1% interest per month. What is the balance in dollars at a point $t$ months after the initial deposit?

Let $B(t)$ represent that balance in dollars.

Let $a$ represent the original balance.

Let $b$ represent the factor by which the balance changes every month. 1% interest means that the account balance grows by a factor of 1.01 each month, if no deposits or withdrawals are made.

Being that $y = a(1 + r)^t$: The equation for the function is $y = 1000(1.01)^t$

10. Write an exponential growth function, and state the domain and range. Tell what the $y$-intercept represents. Sara sold a coin for $3, and its value increases by 2% each year after it is sold. Find the value of the coin in 8 years.
Whenever we repeatedly decrease an amount by a percentage, it’s a “decay” situation.

If \( r \) represents the decay rate and \( t \) represents the time that has passed:

We convert the Standard Exponential Form: \( f(x) = a \cdot b^x \) to the Exponential Decay Model:
\[
y = a \left(1 - r\right)^t
\]
These two forms are essentially the same.

The population of a town is decreasing at a rate of 3% per year. In 2005, there were 1600 people. Find the population in 2013.

Write the exponential decay function for this situation.
\[
y = a(1 - r)^t
\]
\[
= 1600(1 - 0.03)^t
\]
\[
= 1600(0.97)^t
\]

Find the value in 8 years.
\[
y = 1600(0.97)^8
\]
\[
\approx 1254
\]

After 8 years, the town’s population will be about 1254 people.
\[ y = 1600(0.97)^t \]

Create a table of values to graph the function.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( y )</th>
<th>((t, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1600</td>
<td>(0, 1600)</td>
</tr>
<tr>
<td>8</td>
<td>1254</td>
<td>(8, 1254)</td>
</tr>
<tr>
<td>16</td>
<td>983</td>
<td>(16, 983)</td>
</tr>
<tr>
<td>24</td>
<td>770</td>
<td>(24, 770)</td>
</tr>
<tr>
<td>32</td>
<td>604</td>
<td>(32, 604)</td>
</tr>
</tbody>
</table>

Determine the domain and range of the function.

The domain is the set of real numbers \( t \) such that \( t \geq 0 \). The range is the set of real numbers \( y \) such that \( 0 \leq y \leq 1600 \).

The \( y \)-intercept is the value of \( y \) when \( t = 0 \), the number of people before it started to lose population.
The value of a car is depreciating at a rate of 5% per year. In 2010, the car was worth $32,000. Find the value of the car in 2013.

Write the exponential decay function for this situation.

\[ y = a(1 - r)^t \]

Find the value in 3 years.

\[ y = a(1 - r)^t \]

After 3 years, the car’s value will be $\underline{}$. 
Create a table of values to graph the function.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$y$</th>
<th>$(t, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32,000</td>
<td>$(0, 32,000)$</td>
</tr>
<tr>
<td>1</td>
<td>30,400</td>
<td>$(1, 30,400)$</td>
</tr>
<tr>
<td>2</td>
<td>28,880</td>
<td>$(2, 28,880)$</td>
</tr>
<tr>
<td>3</td>
<td>27,436</td>
<td>$(3, 27,436)$</td>
</tr>
</tbody>
</table>

Determine the domain and range of the function.

The domain is the set of real numbers $t$ such that $t \geq 0$. The range is the set of real numbers $y$ such that $0 \leq y \leq 32,000$.

The $y$-intercept, 32,000, is the value of $y$ when $t = 0$, the original value of the car.
A pharmaceutical company is testing a new antibiotic. The number of bacteria present in a sample when the antibiotic is applied is 100,000. Each hour, the number of bacteria present decreases by a third. The number of bacteria remaining \( r(n) \) is an exponential function of the number \( n \) of hours since the antibiotic was applied.

\[
a = \quad b = \quad r(n) =
\]

A laser beam with an output of 5 milliwatts is directed into a series of mirrors. The laser beam loses 1% of its power every time it reflects off of a mirror. The power \( p(n) \) is a function of the number \( n \) of reflections.

\[
a = \quad b = \quad p(n) =
\]
\[ y = a (1 \pm r)^t \]

There are 4 variables: \( y, a, r, \) and \( t \). Given any 3, solve for the 4th.

For example:

**We had this one before:**
A painting is sold for \( $1800 \) (\( a \)), and it increases in value 11% (\( r \)) per year. What’s the value (\( y \)) after 30 (\( t \)) years?
Given (\( a \)), (\( r \)), and (\( t \)), solve for (\( y \)).

**Here are three variations:**
A painting increases in value 11% (\( r \)) per year after it’s sold. If its value after 30 (\( t \)) years is \( $41,206.13 \) (\( y \)), what was its initial value (\( a \))?
Given (\( y \)), (\( r \)), and (\( t \)), solve for (\( a \)).

A painting is sold for \( $1800 \) (\( a \)), and it increases in value each year. If it’s worth \( $41,206.13 \) (\( y \)) after 30 (\( t \)) years, how much did it increase in value each year?
Given (\( a \)), (\( y \)), and (\( t \)), solve for (\( r \)).

A painting is sold for \( $1800 \) (\( a \)), and it increases in value 11% (\( r \)) per year. How many years (\( t \)) will it take for its value to be \( $41,206.13 \) (\( y \))?
Given (\( a \)), (\( r \)), and (\( y \)), solve for (\( t \)).
An example where we solve for \( t \):

An animal reserve has 20,000 elk. The population is increasing at a rate of 8% per year. There is a concern that food will be scarce when the population has doubled. How long will it take for that to happen?

20,000 is the initial amount.
8% is the amount of increase. In decimal, that’s 0.08
So the exponential function is: \( y = 20,000 \ (1 + 0.08)^t \)
   Simplified is: \( y = 20,000 \ (1.08)^t \)

\( y \) is the final amount, which is 40,000.

To solve: \( 40,000 = 20,000 \ (1.08)^t \)
Divide both sides by 20,000: \( 2 = (1.08)^t \)
1.08 to what power is 2?
   Solve: \( t = 9 \) years
Two more examples where we solve for the $t$:

A lake has a small population – 10 – of a rare endangered fish. Its number is increasing at a rate of 4% per year. How long will it take the population to be 80 fish?

10 is the initial amount.
4% is the amount of increase. In decimal, that’s 0.04
So the exponential function is: $y = 10 \cdot (1 + 0.04)^t$
Simplified is: $y = 10 \cdot (_______)^t$
To solve: $80 = 10 \cdot (_______)^t$
Divide both sides by 8: $8 = (_______)^t$
Solve: $t = 53$ years

There is a drought and the 50,000 oak trees are decreasing at the rate of 7% per year. How long will it take for the population to be reduced by half?

50,000 is the initial amount.
7% is the amount of increase. In decimal, that’s 0.07
So the exponential function is: $y = 50,000 \cdot (1 - 0.07)^t$
Simplified is: $y = 50,000 \cdot (_______)^t$
To solve: $50,000 = 50,000 \cdot (_______)^t$
Divide both sides by 50,000: $1 = (0.93)^t$
Solve: $t = \text{between } \underline{______} \text{ and } \underline{______} \text{ years}$
Write an exponential function to model each situation. Then find the value of the function after the given amount of time.

Annual sales for a company are $155,000 and increases at a rate of 8% per year for 9 years.

The value of a textbook is $69 and decreases at a rate of 15% per year for 11 years.

Describe three real-world situations that can be described by exponential growth or exponential decay functions. **Possible answers:** interest earned on an investment, population growth or decline, radioactive decay

**Connect Vocabulary**
Remind students that exponential growth refers to an increasing function and exponential decay refers to a decreasing function.

Examples of key words for growth include increases, goes up, rises, gains.

Examples of key words for decay include decreases, goes down, falls, loses value, declines, depreciates.

**AVOID COMMON ERRORS**
Some students may forget to convert the percent growth rate to decimal form. Remind them that growth rate must be written as a decimal because the percent sign means “parts out of 100.”
Write an exponential function to model each situation. Then find the value of the function after the given amount of time.

5. Annual sales for a company are $155,000 and increases at a rate of 8% per year for 9 years.

6. The value of a textbook is $69 and decreases at a rate of 15% per year for 11 years.

7. A new savings account is opened with $300 and gains 3.1% yearly for 5 years.

8. The value of a car is $7800 and decreases at a rate of 8% yearly for 6 years.

9. The starting salary at a construction company is fixed at $55,000 and increases at a rate of 1.8% yearly for 4 years.

10. The value of a piece of fine jewelry is $280 and decreases at a rate of 3% yearly for 7 years.
Comparing Exponential Growth and Decay

Graphs can be used to describe and compare exponential growth and exponential decay models over time.

**Example 3**  Use the graphs provided to write the equations of the functions. Then describe and compare the behaviors of both functions.

The graph shows the value of two different shares of stock over the period of 4 years since they were purchased. The values have been changing exponentially.

The graph for Stock A shows that the value of the stock is decreasing as time increases.

The initial value, when \( t = 0 \), is 16. The value when \( t = 1 \) is 12. Since \( 12 \div 16 = 0.75 \), the function that represents the value of Stock A after \( t \) years is \( A(t) = 16(0.75)^t \). \( A(t) \) is an exponential decay function.

The graph for Stock B shows that the value of the stock is increasing as time increases.

The initial value, when \( t = 0 \), is 2. The value when \( t = 1 \) is 3. Since \( 3 \div 2 = 1.5 \), the function that represents the value of Stock B after \( t \) years is \( B(t) = 2(1.5)^t \). \( B(t) \) is an exponential growth function.

The value of Stock A is going down over time. The value of Stock B is going up over time. The initial value of Stock A is greater than the initial value of Stock B. However, after about 3 years, the value of Stock B becomes greater than the value of Stock A.